A Theory of the Cross-Sectional Fertility Differential: Jobs' Heterogeneity Approach

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Abstract

This paper presents a theory of the cross-sectional fertility differential, which produces the negative wage-fertility relationship based on jobs' heterogeneity. Compared to the existing literature, the theory not only captures the realistic situation where productivity and working conditions differ across jobs, but also requires only standard conditions on preferences to generate the negative relationship. Moreover, the result is robust to changes in economic environments (e.g., public policy and technology). The theory reconciles the negative cross-sectional wage-fertility relationship with various time-series variation in the aggregate fertility. Furthermore, this study adds an important viewpoint to the empirical literature.

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1 Introduction

This paper presents a tractable theory of the cross-sectional fertility differential, which produces the negative relationship between wages and fertility based on the heterogeneity of job productivity. Since the seminal work of Gary Becker (Becker, 1960), a large number of studies have developed fertility theories by using microeconomic techniques. This paper introduces new insights into the literature. In contrast to the existing literature, which focuses on heterogeneity of the individual ability, we focus on heterogeneity of job (or firm) productivity.[1] To introduce the jobs' heterogeneity, we construct a labor market model where firms offer jobs specifying wages and working hours, individuals apply to preferable jobs, and one job and one individual form a production unit. This is in contrast with the rest of the literature, which conventionally assumes that individuals facing given per-time wages (or given wages per unit of human capital) allocate their time between labor supply and child-rearing. In our model, the cross-sectional fertility differential arises as an equilibrium outcome in the competitive labor market, where jobs with different productivity levels offer different labor contracts.[2] Compared to existing theories, our theory captures the realistic situation where productivity and working conditions differ across jobs and obtains the result that fertility negatively correlates with wages under moderate restrictions on preferences. Moreover, the result is robust to changes in economic environments (e.g., public policy and technology).

A widespread finding in the economics of fertility is the negative cross-sectional relationship between wages and fertility: wealthier parents tend to have fewer children.[3] A large part of theoretical literature has attempted to explain this negative relationship without assuming that children are an inferior good. The assumption commonly used in the literature is that the

[1] Jones, Schoonbroodt and Tertilt (2011) provide an excellent overview of existing theories in the literature.

[2] Since the structure of our model considerably differs from that employed by other studies, one may argue that the results obtained in this paper are attributable to the special structure, but it is incorrect. The structure is just an assumption for introducing jobs' heterogeneity. It is the introduction of jobs' heterogeneity that departs our results from those obtained in the literature so far. See Section 5 for details.

[3] The negative relationship between wages and fertility is widespread across time and regions, but some exceptional findings are reported. See Section 5 for details.

cost of children is largely parents' time, and because of this, parents with higher wages face a higher price of children. On the other hand, parents with higher wages have more wealth. The usual substitution and wealth effects coexist: if the substitution effect dominates the wealth effect, wealthier parents have fewer children, and vice versa. Thus, the literature conventionally imposes restrictive assumptions on preferences that diminish the wealth effect relative to the substitution effect in order to explain the negative effect of wages on fertility. Alternatively, not a few studies in the literature add the parental choice on child's quality to derive the negative relationship: wealthier parents want more quality and thus less quantity. Adding the quality choice by itself, however, does not generate the negative relationship, and additional restrictive assumptions on preferences and/or the child-quality production function are needed (see, for instance, Becker and Tomes (1976) and Moav (2005)).

By using the US data on 30 birth cohorts between 1830 and 1960, Jones and Tertilt (2008) find that the negative cross-sectional relationship has been surprisingly stable. Over the same period, technological progress has led to a rise in real wages, the educational level of the population has risen, the government has conducted various policies (e.g., tax reform, education reform, and social-security reform), and there has been a long secular decline in the average fertility rate, interrupted by a temporary rise (i.e., a baby boom). In other words, in the last two centuries of the United States, the cross-sectional relationship between wages and fertility remained negative despite considerable changes in economic environments and time-series variations in the aggregate fertility. However, with few exceptions, the negative relationship derived from existing theories are not robust to changes in assumptions, for example, changes in the form of preferences, the form of education function, and the type and the size of policy variables. This paper is intended to construct a model in which the negative cross-sectional relationship is produced without imposing restrictive assumptions on preferences and is robust to changes in economic environments.

The only crucial assumption in our model is that it takes time to raise children. Individuals prefer higher consumption (i.e., higher wages) and higher fertility. By the assumption that child rearing takes time, longer working hours mean lower fertility. Thus, to induce individuals to work longer hours, firms must pay higher wages to compensate lower fertility. Firms are faced with this tradeoff in offering labor contracts, and firms with higher-productivity job want their employees to work longer hours even if they must pay higher rewards because those jobs can produce the larger quantity of output per unit time. In contrast to the conventional fertility model based on individuals' heterogeneity, our model does not entail welfare differential among individuals because individuals with different wages merely choose different contracts on the same indifference curve: there is only the substitution effect, not the wealth effect. Therefore, our model needs no restriction on the relative size between the substitution and wealth effects to generate the negative cross-sectional wage-fertility relationship.

Jones, Schoonbroodt and Tertilt (2011) share with us the motivation to produce a robust negative cross-sectional relationship between wages and fertility, and they present a prospective theory. In contrast with our theory, which focuses on the heterogeneity of job productivity, theirs sheds light on the heterogeneity of individual preferences. They attribute the negative relationship to unobserved heterogeneity in parental preferences for fertility. Persons who have a preference for children receive less education in anticipation of having more children and supplying less labor to the market in the future, and as a result, they actually earn lower wages and have more children.[4] The causality is opposite to the conventional fertility theory, from fertility to parental wages. The negative relationship derived from their theory is quite robust to changes in the form of preferences: it does not depend on specific functional forms or parameter restrictions. Our theory is as robust as theirs with respect to changes in the form of preferences. An advantage of our theory is that we do not even need to introduce education into the model. Even in the United States, in the 19th century, there was limited scope for human capital investments based on one's own choice, but the negative relationship was observed (Jones and Tertilt, 2008). Our theory is applied to the economy where the free choice of employment is guaranteed, even

[4]Kimura and Yasui (2007) also introduce parents' own educational choice to derive the cross-sectional negative wage-fertility relationship. In Kimura and Yasui (2007), the source of education differential is not taste heterogeneity, but the arbitrage between becoming skilled or remaining unskilled.

if the education system is underdeveloped there.

Mookherjee, Prina and Ray (2012) have a similar motivation. They try to produce a robust negative cross-sectional relationship between wages and fertility based on a dynamic general equilibrium approach. In their model, choice between skilled and unskilled occupations by parents plays an essential role in producing the negative relationship.[5] Their breakthrough is to restrict individuals' fertility behavior by using the steady-state condition of the dynamic general equilibrium, not imposing restrictions on preferences itself. Based on the dynamic general equilibrium model, however, they inevitably rely on the specification of preferences to some degree: although they impose no restriction on the elasticity of substitution between consumption and fertility, they impose a restrictive assumption on the elasticity of substitution between fertility and child quality.[6]

There has been a recent increase in economic theories endogenizing fertility choice.[7] One broader aim of this paper is to offer an "off-the-shelf" fertility model used as one of the building blocks in applied theories. In addition to the tractability, our theory has a useful characteristic as a building block: it can reconcile the negative cross-sectional wage-fertility relationship with various time-series variation in the aggregate fertility. Based on our model, the driving force behind the aggregate fertility variation across time can be separated from that behind the cross-sectional fertility differential. It is possible, for instance, that the dominant wealth effect associated with changes in overall productivity and public policy raises the aggregate fertility whereas the cross-sectional relationship remains negative. Especially in the macroeconomics literature, the impacts of various policy changes are analyzed (e.g., Greenwood, Guner and

[5] Doepke (2004) uses a similar mechanism. The result of Mookherjee, Prina and Ray (2012) is a generalization of Doepke (2004) to the case in which restrictions on preferences concerning the relative magnitude of wealth and substitution effects are relaxed.

[6] Another merit of the theory of Mookherjee, Prina and Ray (2012) is that it can explain the widespread negative correlation between wages and fertility, while allowing for exceptions that arise within occupations. Our theory also presents a new insight on this point. See Section 5 for details.

[7] For instance, there are studies on the demographic transition and the long-run growth (e.g., Galor and Weil (1996), Galor and Weil (2000), Galor and Moav (2002), and Hansen and Prescott (2002)), those on the international differences in fertility (e.g., Adsera (2004) and Manuelli and Seshadri (2009)), and those on the baby boom following World War II (e.g., Greenwood, Seshadri and Vandenbroucke (2005), Doepke, Hazan and Maoz (2007), Jones and Schoonbroodt (2010), and Kimura and Yasui (2010)).

Knowles (2003), Boldrin, De Nardi and Jones (2005), Zhao (2009), Erosa, Fuster and Restuccia (2010)). In such studies, a careful decision about which one to use as a building block is needed because for the results to be consistent with data, at least on the last two centuries of the United States, the negative wage-fertility relationship must be preserved in response to various policy changes. Our theory satisfies this criterion as a building block: without imposing restrictive assumptions on preferences, various types of policies can be introduced and their sizes can be changed flexibly.

The remainder of this paper is organized as follows. Section 2 presents the basic model and conducts some comparative-static analyses. Section 3 provides an example, where functional forms are specified and a closed-form solution is derived. Section 4 extends the model to include quantity-quality tradeoff and market childcare. In Section 5, we discuss the relationship between the conventional fertility theory and our theory. Section 6 concludes this paper.

2 Model

2.1 Structure

Individuals The economy is populated by a continuum of measure N of ex ante homogeneous individuals. All individuals are assumed to have the same strictly increasing, strictly quasi-concave, twice continuously differentiable utility function $u(n,c)$ over the number of children, n, and consumption, c. We omit leisure from the model for simplification, but the introduction of leisure does not undermine our advantage in the robustness of the negative wage-fertility relationship over existing theories (see Footnote 10 for details) The indifference curve in the (n,c) plane corresponding to utility level U can be written as

$$c = \Psi_U(n), \tag{1}$$

which satisfies

$$\frac{\partial \Psi_U(n)}{\partial n} < 0,\ \frac{\partial^2 \Psi_U(n)}{\partial n^2} > 0, \text{ and } \frac{\partial \Psi_U(n)}{\partial U} > 0.$$

Throughout this paper, functions and variables with the subscript "U" represent those associated with the utility level U. We further add an Inada-type condition to guarantee that all the time available is not devoted to work:

$$\lim_{n \to 0} \frac{\partial \Psi_U(n)}{\partial n} = -\infty.$$

This assumption is only for simplification, not essential to the model. These are all the restrictions imposed on preferences to derive the negative cross-sectional wage-fertility relationship. We impose no further restriction on the curvature of utility function, as well as the elasticity of substitution between fertility and consumption.

The total time endowment of an individual is normalized to 1. Producing a child takes $\tau > 0$ units of time. We let $l \in [0,1]$ and $w \geq 0$ denote the time spent working and the labor income, respectively. The time constraint and the budget constraint are respectively given by

$$\tau n + l = 1 \tag{2}$$

and

$$(1+t_c)\,c = (1-t_w)\,w + \theta + s_n n, \tag{3}$$

where $t_c \geq 0$, $t_w \in [0,1)$, $s_n \geq 0$, and $\theta \geq 0$ are the consumption tax rate, the labor-income tax rate, the child-rearing subsidy, and the non-labor income, respectively. To demonstrate the robustness of our theory, we include various variables, some of which are often omitted in the literature. It should be especially noted that a negative child-rearing subsidy is the same as the goods costs of childcare for parents. As will become apparent, our main result (Proposition 1) does not depend on the value of s_n, whether positive or negative, implying that our theory is robust to the inclusion of goods costs of childcare.[8]

[8] A negative non-labor income is the same as a lump-sum tax for parents. The value of θ, whether positive

Jobs There is a large pool of prospective entrants into the market. Prior to entry, firms are identical. To open a vacant job, firms must incur an initial investment cost $\kappa > 0$, which is thereafter sunk. Vacant jobs then draw their productivity parameter α from a common distribution $F(\cdot)$. It is assumed that the support of $F(\cdot)$ is $\mathbb{R}_+$ and that $F(\cdot)$ is strictly increasing and twice continuously differentiable. One job and one individual form a production unit. When an individual works in a job with productivity α, the match produces α units of good per unit time. If a job with productivity α hires a worker on the condition that working hours is l and compensation is w, the (after-tax) profit is

$$\Pi(\alpha) = (1 - t_\Pi)(\alpha l - w), \tag{4}$$

where $t_\Pi \in [0, 1)$ is the corporate-profit tax rate. Note that after the entry, the initial investment cost was sunk.[9]

Labor market Individuals and vacancies come together in the competitive labor market: each vacancy offers a contract specifying working hours, l, and compensation, w, and each individual freely applies to a vacancy to work. There are no frictions in the matching process.

Timing of events The timing of events is as follows.

1. Each firm decides whether or not to open a vacancy with a fixed cost κ.

2. All vacancies draw their productivity α from a common distribution $F(\cdot)$.

3. Each vacancy offers a contract specifying working hours, l, and compensation, w, and each individual observes all posted contracts and decides on which of these to apply freely.

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or negative, does not affect our main result (Proposition 1), implying that our theory is robust with respect to the inclusion of lump-sum tax. Furthermore, we assume the linear child-rearing cost, τn, but we do not need this assumption to derive the main result, which does not depend on whether child production is decreasing, increasing, or constant returns to scale.

[9] In reality, investment cost is at least partially deductible in many countries. Even if we assume that investment spending is tax deductible, our results do not change qualitatively except for the comparative-static analysis on the corporate-profit tax rate.

4. All matches implement their contracts. Individuals work a promised schedule and production takes place. Firms pay promised earnings to employees. Further, each individual raises children during out-of-work hours.

2.2 Competitive equilibrium

In this subsection, we derive a competitive equilibrium of this economy. An allocation is a tuple $\{\underline{\alpha}, \{w(\alpha)\}_{\alpha=\underline{\alpha}}^{\infty}, \{l(\alpha)\}_{\alpha=\underline{\alpha}}^{\infty}, U, \{\Pi(\alpha)\}_{\alpha=\underline{\alpha}}^{\infty}, p, M\}$, where $\underline{\alpha}$ is the threshold productivity below which jobs do not operate, $w(\alpha)$ is compensation in jobs with productivity α, $l(\alpha)$ is working hours in jobs with productivity α, U is the individuals' utility level, $\Pi(\alpha)$ is the profit of jobs with productivity α, p is the proportion of individuals employed, and M is the measure of entrants. We now define a competitive equilibrium.

Definition 1. *A competitive equilibrium is* $\{\underline{\alpha}^*, \{w^*(\alpha)\}_{\alpha=\underline{\alpha}^*}^{\infty}, \{l^*(\alpha)\}_{\alpha=\underline{\alpha}^*}^{\infty}, U^*,$ $\{\Pi^*(\alpha)\}_{\alpha=\underline{\alpha}^*}^{\infty}, p^*, M^*\}$*, which satisfies the following conditions.*

1. *All individuals optimally choose their jobs (or not to be employed) and have the same utility level.*

2. *Each job chooses working hours and compensation (or not to post any labor contract) to maximize the profit.*

3. *The labor market clears: the measure of employed individuals is equal to the measure of active jobs.*

4. *Firms freely open vacancies: firms continue to open vacancies until the expected profit is driven to the initial investment cost.*

We solve the model in two steps. In the first step, given the utility level of individuals, U, we derive the equilibrium allocation $\{\underline{\alpha}, \{w(\alpha)\}_{\alpha=\underline{\alpha}}^{\infty}, \{l(\alpha)\}_{\alpha=\underline{\alpha}}^{\infty}, \{\Pi(\alpha)\}_{\alpha=\underline{\alpha}}^{\infty}\}$ as a function of U. In the second step, given the result of the first step, we derive the equilibrium allocation

$\{U,\, p, M\}$ by using the labor-market clearing and free-entry conditions. Then, combining them, we arrive at the competitive equilibrium allocation.

First step

Let us start with the first step. Since each firm is small relative to the market size and has no market power, it takes the utility level of individuals as given and must post a contract guaranteeing the utility level to attract them. Properties of individuals' induced indifference curves in the (l,w) plane are central in considering labor contracts posted by firms. In Lemma 1 below, we demonstrate that an indifference curve mapping in the (l,w) plane is as shown in Figure 1.

Lemma 1. *(a) The induced indifference curve in the (l,w) plane corresponding to utility level U, $w = \Omega_U(l)$, satisfies*

$$\frac{\partial \Omega_U(l)}{\partial l} > 0,\ \frac{\partial^2 \Omega_U(l)}{\partial l^2} > 0,\ \frac{\partial \Omega_U(l)}{\partial U} > 0, \text{ and } \lim_{l \to 1} \frac{\partial \Omega_U(l)}{\partial l} = \infty.$$

(b) The bottom curve passes through the origin.

Proof. (a) Substituting (1) and (2) into (3), we obtain

$$w = \frac{1}{1-t_w}\left[(1+t_c)\,\Psi_U\left(\frac{1-l}{\tau}\right) - \theta - s_n \frac{1-l}{\tau}\right] \equiv \Omega_U(l)\,, \tag{5}$$

which is the induced indifference curve in the (l,w) plane corresponding to utility level U. Using the property of $\Psi_U(n)$, we establish that

$$\frac{\partial \Omega_U(l)}{\partial l} = \frac{1}{(1-t_w)\,\tau}\left[-(1+t_c)\frac{\partial \Psi_U(n)}{\partial n} + s_n\right] > 0, \tag{6}$$

$$\frac{\partial^2 \Omega_U(l)}{\partial l^2} = \frac{1+t_c}{(1-t_w)\,\tau^2}\frac{\partial^2 \Psi_U(n)}{\partial n^2} > 0,$$

$$\frac{\partial \Omega_U(l)}{\partial U} = \frac{1+t_c}{1-t_w}\frac{\partial \Psi_U(n)}{\partial U} > 0,$$

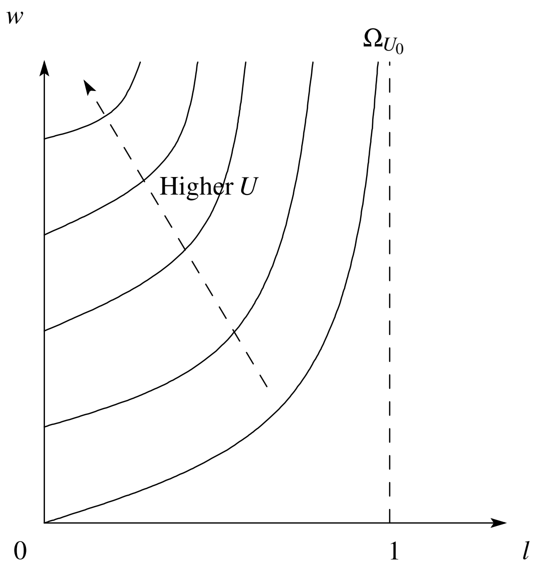


Figure 1: Induced indifference curves in (l,w) plane.

and

$$\lim_{l\to 1}\frac{\partial\Omega_U(l)}{\partial l}=\lim_{n\to 0}\frac{1}{(1-t_w)\,\tau}\left[-(1+t_c)\frac{\partial\Psi_U(n)}{\partial n}+s_n\right]=\infty.$$

(b) Since individuals can choose not to be employed, their utility level is bounded below by U_0, which is the utility level associated with $l=0$ and $w=0$. Thus, U_0 is given by

$$\Omega_{U_0}(0)=0, \tag{7}$$

and $w=\Omega_{U_0}(l)$ passes through the origin. □

The induced indifference curve in the (l,w) plane is upward sloping and convex to the l axis. The former property is derived from the fact that u is increasing in both arguments: to be indifferent, lower fertility (i.e., longer working hours) must be compensated by higher consumption (i.e., higher pay). The latter property is derived from quasi concavity of u: since the marginal rate of substitution of c for n increases as (n,c) moves toward the northwest along an indifference curve, $c=\Psi_U(n)$, the increment of consumption (i.e., compensation) required to induce individuals to reduce fertility (i.e., work extra hours) increases as working hours and compensation increase. The induced indifference curve associated with higher U is located above the one associated with lower U. Obviously, shorter working hours and higher compensation are associated with higher utility.

Given U, vacancies post labor contracts maximizing their profits within the constraint of $w \geq \Omega_U(l)$. A vacancy with productivity α solves the following maximization problem:

$$\max_{l \in [0,1], w \geq 0} \Pi(\alpha) = (1 - t_\Pi)(\alpha l - w),$$

$$\text{s.t. } w \geq \Omega_U(l).$$

Solving this problem leads us to the following proposition.

Proposition 1. *A job with higher productivity offers a labor contract with longer working hours and higher compensation. An employee in a higher-productivity job has fewer children.*

Proof. Since the constraint $w \geq \Omega_U(l)$ holds with equality, we can rewrite the job's problem as

$$\max_{l \in [0,1]} \Pi_U(\alpha) = (1 - t_\Pi)[\alpha l - \Omega_U(l)].$$

The solution to this problem can either be interior or at a corner; there is a threshold level of productivity, $\tilde{\alpha}_U$, below which jobs maximize their profits by setting $l = 0$. The threshold level is

$$\tilde{\alpha}_U = \frac{\partial \Omega_U(0)}{\partial l} > 0.$$

The solution is

$$\begin{cases} \alpha - \frac{\partial \Omega_U(l)}{\partial l} = 0 & \text{if} \quad \alpha > \tilde{\alpha}_U, \\ \\ l = 0 & \text{if} \quad \alpha \leq \tilde{\alpha}_U. \end{cases} \tag{8}$$

The case of $l = 1$ is excluded because $\lim_{l \to 1}[\partial \Omega_U(l)/\partial l] = \infty$. Totally differentiating the first-

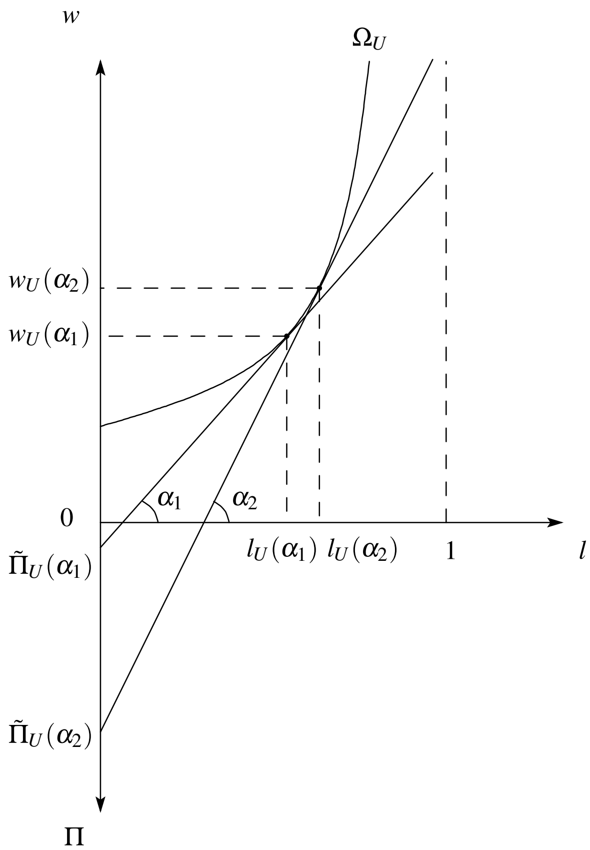


Figure 2: Labor contracts offered by jobs with α_1 and α_2 ($> \alpha_1$).

order condition of the interior solution, we obtain

$$l_U'(\alpha) = \left[\frac{\partial^2 \Omega_U(l_U(\alpha))}{\partial l_U(\alpha)^2}\right]^{-1} > 0. \tag{9}$$

The sign comes from Lemma 1. It follows from (2) and (5) that this result implies $n_U'(\alpha) < 0$ and $w_U'(\alpha) > 0$. $\square$

This proposition is the main result of this paper. If jobs with different productivity levels coexist, then we observe a negative cross-sectional relationship between labor income and fertility. The intuition is simple. For individuals to be indifferent, longer working hours must be compensated by higher rewards. Firms are faced with this tradeoff in posting labor contracts, and firms with higher-productivity jobs want their employees to work longer hours even if they must pay higher rewards because those firms can produce a larger quantity of output per unit time. Figure 2 graphically illustrates this argument. Individuals' induced indifference curves in the (l, w) plane are as shown in Figure 1. Equation (4) implies that jobs' iso-profit curves are upward sloping and the slope, α, is steeper for higher-productivity jobs. If a higher-productivity job decreases labor inputs, l, then the output, αl, decreases by the larger amount. Thus, to keep the profit of such a job constant, compensation, w, must decrease by the larger amount. A posted contract is a point of tangency between these two curves, which is unique. It follows that a higher-productivity job chooses longer working hours (implying fewer children) and higher compensation ($w_U(\alpha_1) < w_U(\alpha_2)$ and $l_U(\alpha_1) < l_U(\alpha_2)$, where $\alpha_1 < \alpha_2$).[10] Figure 2 also depicts the pre-tax profits associated with each labor contract, $\tilde{\Pi}_U(\alpha) \equiv \Pi_U(\alpha) / (1 - t_\Pi) = \alpha l_U(\alpha) - w_U(\alpha)$.

Although the above analysis focuses on the relationship between *total* pay and fertility, it is customary in the literature to analyze the relationship between *per-time* pay and fertility. We check whether our argument about the relationship between total pay and fertility also holds for the relationship between per-time pay and fertility. Differentiating the per-time pay with respect to α and using (4), (8), and (9), we obtain

$$\frac{\partial}{\partial \alpha}\left[\frac{w_U(\alpha)}{l_U(\alpha)}\right] = \frac{\partial}{\partial \alpha}\left[\frac{\Omega_U(l_U(\alpha))}{l_U(\alpha)}\right] = \frac{l_U'(\alpha)\,\Pi_U^*(\alpha)}{(1 - t_\Pi)\,l_U(\alpha)^2} > 0,$$

where $\Pi_U^*(\alpha)$ is the maximized profit of a job with productivity α for given U. The maximized profit is positive because jobs whose profit is negative do not operate. It follows that the statement of Proposition 1 regarding the relationship between total pay and fertility also holds for the relationship between per-time pay and fertility: *a job with higher productivity offers a labor contract with longer working hours and higher per-time pay*. This comes from strict quasi concavity

[10] The negative wage-fertility relationship does not crucially depend on the assumption that child-rearing time is the mirror image of labor supply. The negative relationship derived from our model is more robust to the introduction of the third time use (e.g., leisure) than that derived from individuals' heterogeneity approach. Even if we introduce leisure into the model, the result that longer working hours must be compensated by higer pay does not change. Both child-rearing time and leisure decrease as working hours increases unless the complementarity between consumption and leisure is sufficiently large compared to the complementarity between consumption and children.

of u. The increment of earnings required to induce individuals to work an extra hour increases as working hours and pay increase. Since the marginal compensation required increases with working hours, so does the average compensation.

The productivity level $\tilde{\alpha}_U$ separates jobs whose profit is maximized at $l > 0$ from those whose profit is maximized at $l = 0$. However, all jobs with $\alpha > \tilde{\alpha}_U$ do not necessarily operate because the maximized profit might be negative even for such jobs. Since firms can freely choose whether to operate or not, only jobs with positive maximized profit offer labor contracts. Taking this into consideration, we can derive the condition for jobs to operate.

Lemma 2. *There exists a threshold productivity level $\underline{\alpha}_U$, below which jobs do not operate:*

(a) $\underline{\alpha}_U \geq \tilde{\alpha}_U$ for all $U \geq U_0$, with strict inequality if $U > U_0$, and

(b) $\underline{\alpha}_U$ increases with U.

Proof. Given the utility level of individuals, U, the maximized profit of a job with productivity α is

$$\Pi_U^*(\alpha) = (1 - t_\Pi)\left[\alpha l_U(\alpha) - \Omega_U(l_U(\alpha))\right], \tag{10}$$

where $l_U(\alpha)$ is given by (8). The productivity level that makes jobs indifferent between operating or not, $\underline{\alpha}_U$, is given by $\Pi_U^*(\underline{\alpha}_U) = 0$. It follows from this and (8) that

$$\underline{\alpha}_U = \frac{\Omega_U(l_U(\underline{\alpha}_U))}{l_U(\underline{\alpha}_U)} = \frac{\partial \Omega_U(l_U(\underline{\alpha}_U))}{\partial l}. \tag{11}$$

For the threshold job, the per-time compensation, Ω/l, and the marginal rate of substitution between l and w, $\partial\Omega/\partial l$, are equalized to the productivity level. Since Ω is strictly convex to the l axis,

$$\frac{\partial \Omega_U(l)}{\partial l} > \frac{\partial \Omega_U(0)}{\partial l} \text{ for all } l > 0.$$

If $U > U_0$, then

$$\underline{\alpha}_U = \frac{\Omega_U(l_U(\underline{\alpha}_U))}{l_U(\underline{\alpha}_U)} = \frac{\partial \Omega_U(l_U(\underline{\alpha}_U))}{\partial l} > \frac{\partial \Omega_U(0)}{\partial l} = \tilde{\alpha}_U.$$

If $U = U_0$, then $w = \Omega_U(l)$ passes through the origin, and thus

$$\underline{\alpha}_{U_0} = \lim_{l_U(\underline{\alpha}_U)\to 0} \frac{\Omega_{U_0}(l_U(\underline{\alpha}_U))}{l_U(\underline{\alpha}_U)} = \frac{\partial \Omega_{U_0}(0)}{\partial l} = \tilde{\alpha}_{U_0}.$$

By the envelope theorem, we obtain

$$\frac{\partial \Pi_U^*(\alpha)}{\partial \alpha} = (1 - t_\Pi)\, l_U^*(\alpha) > 0,$$

and

$$\frac{\partial^2 \Pi_U^*(\alpha)}{\partial \alpha^2} = (1 - t_\Pi)\, l_U^{*\prime}(\alpha) > 0.$$

Therefore, the threshold productivity level $\underline{\alpha}_U$ is unique and exists in $[\tilde{\alpha}_U, \infty)$. Differentiating $\Pi_U^*(\alpha)$ with respect to U, we obtain

$$\frac{\partial \Pi_U^*(\alpha)}{\partial U} = -(1 - t_\Pi)\frac{\partial \Omega_U(l_U(\alpha))}{\partial U} < 0.$$

The maximized profit of a job with productivity α, $\Pi_U^*(\alpha)$, decreases with U for all α. Therefore, the threshold productivity level increases with U. $\square$

Since the maximized profit increases with α, there exists a threshold productivity, below which jobs do not operate. We can graphically confirm this by investigating Figure 2. The (pre-tax) profit of jobs whose iso-profit curve tangent to the relevant induced indifference curve passes the origin is zero, and those jobs are the exit thresholds, that is, $\tilde{\Pi}_U(\underline{\alpha}_U) = 0$. This lemma also states that as the utility level of individuals increases, the ratio of active jobs to all the entrants decreases. Higher utility means that higher compensation is required; thus, the jobs' profits are reduced, driving low-productivity jobs out of the market.

Second step

It follows from (5) and (7) that the value of U_0 is exclusively determined by parameters, not affected by other endogenous variables. Since all individuals must have the same utility level in equilibrium, the utility level is fixed to the outside-option value, U_0, as long as unemployment exists, that is, $p < 1$. Taking this into consideration, we can write the market-clearing condition of the labor market as

$$pN = \begin{cases} [1-F(\underline{\alpha}_{U_0})]M & \text{if } p < 1, \\ \\ [1-F(\underline{\alpha}_{U})]M & \text{if } p = 1. \end{cases} \tag{12}$$

The LHS is the measure of employed individuals and the RHS is the measure of active jobs. Using this market-clearing condition leads us to the following lemma.

Lemma 3. *The utility level of individuals is a nondecreasing function of the measure of entrants:*

$$U'(M) \begin{cases} = 0 & \textit{if} \quad M \leq \tilde{M}, \\ \\ > 0 & \textit{if} \quad M > \tilde{M}. \end{cases}$$

$$\textit{where } \tilde{M} \equiv \frac{N}{1-F(\underline{\alpha}_{U_0})}$$

Proof. In the case of $p < 1$, the values of p and M are not uniquely determined. Since $\underline{\alpha}_{U_0}$ is independent of p and M, all the pairs (p, M) satisfying $pN = [1-F(\underline{\alpha}_{U_0})]M$ are possible. In this range, an increase in the measure of entrants, M, only results in a proportionate increase in the ratio of employed individuals, p, but does not affect the individuals' utility level and the firms' profits at all. Once p reaches 1, on the other hand, since the ratio of employed individuals cannot increase further, in response to an increase in M, $\underline{\alpha}_U$ must increase to keep the measure of active firms constant and balance supply and demand. Therefore, $U(M)$ is constant over the interval $\left[0, N/[1-F(\underline{\alpha}_{U_0})]\right]$ and increasing over $\left(N/[1-F(\underline{\alpha}_{U_0})], \infty\right)$. $\square$

As the measure of entrants increases, the measure of high-productivity jobs (as well as that of low-productivity jobs) increases because the productivity distribution is exogenously given. Owing to the competition among those jobs, the utility level of an individual increases. The utility level continues to increase until the exit threshold becomes high enough for the supply and demand to balance.

We close the model by considering the firms' free entry. Since firms continue to open vacancies until the expected profit is driven to the initial investment cost, the following condition holds:

$$\int_{\underline{\alpha}_U}^{\infty} \Pi_U^* (\alpha) \, dF (\alpha) \leq \kappa,$$

with equality if there are positive entrants. Combining this free-entry condition with Lemmas 2 and 3, we establish the existence of the competitive equilibrium.

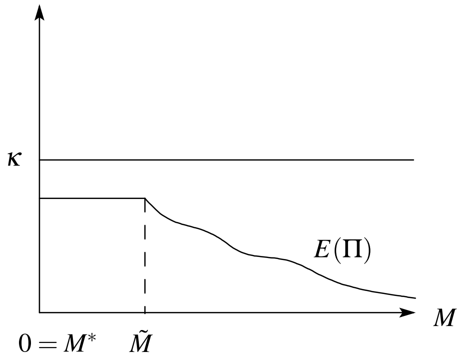
Proposition 2. *There exists the competitive equilibrium, where*

$$M^* \begin{cases} = 0 \quad \text{if} \quad \int_{\underline{\alpha}_U}^{\infty} \Pi_U^* (\alpha) \, dF (\alpha) \Big|_{M=0} \leq \kappa, \\ \\ > 0 \quad \text{if} \quad \int_{\underline{\alpha}_U}^{\infty} \Pi_U^* (\alpha) \, dF (\alpha) \Big|_{M=0} > \kappa. \end{cases} \tag{13}$$

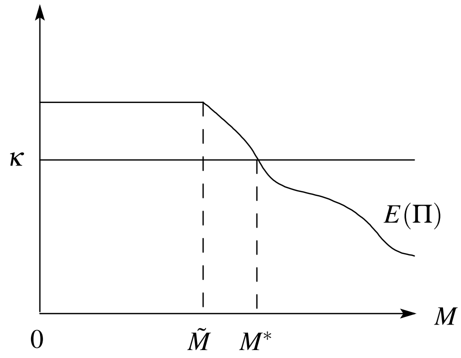
Proof. From Lemma 3, $U(M)$ is constant over $M \in [0, \tilde{M}]$ and increasing over $M \in (\tilde{M}, \infty)$. From Lemma 2, $\underline{a}_U$ increases with U and $\Pi_U^* (\alpha)$ decreases with U for all α. Therefore, $\int_{\underline{\alpha}_U}^{\infty} \Pi_U^* (\alpha) \, dF (\alpha)$ is constant over the interval $M \in [0, \tilde{M}]$ and decreasing over $M \in (\tilde{M}, \infty)$. If $\int_{\underline{\alpha}_U}^{\infty} \Pi_U^* (\alpha) \, dF (\alpha) \leq \kappa$ when $M = 0$, then there is no entry, and thus $M^* = 0$. If $\int_{\underline{\alpha}_U}^{\infty} \Pi_U^* (\alpha) \, dF (\alpha) >$ κ when $M = 0$, then firms continue to open vacancies until $\int_{\underline{\alpha}_U}^{\infty} \Pi_U^* (\alpha) \, dF (\alpha) = \kappa$, and thus $M^* > 0$. □

Figure 3 graphically illustrates this proposition ($E(\cdot)$ represents the expectation over α).[11] The competitive equilibrium is $\{\underline{\alpha}^*, \{w^* (\alpha)\}_{\alpha=\underline{\alpha}^*}^{\infty}, \{l^* (\alpha)\}_{\alpha=\underline{\alpha}^*}^{\infty}, U^*, \{\Pi^* (\alpha)\}_{\alpha=\underline{\alpha}^*}^{\infty}, p^*, M^*\},$

[11] Strictly speaking, if the expected profit evaluated at $M = 0$ is equal to κ, then multiple equilibria arise. However, we do not emphasize the multiplicity because it arises only when the parameters happen to satisfy such a condition, which is practically impossible. Furthermore, it is not robust to changes in assumptions. For instance, if the initial



(a) $M^* = 0$



(b) $M^* > 0$

Figure 3: Equilibrium measure of entrants.

which satisfies (4), (5), (8), (11), (12), and (13). The equilibrium utility level, U^*, depends on functional forms and parameters, but the negative wage-fertility relationship holds for any level of U (Proposition 1), and thus we always observe the negative cross-sectional relationship in equilibrium, independent of functional forms and parameters.

2.3 Comparative statics

In this subsection, we conduct comparative-static analyses on the effects of changes in policy variables (consumption tax rate, t_c, labor-income tax rate, t_w, child-rearing subsidy, s_n, and corporate-profit tax rate, t_Π) and the effect of technological change. For the analysis, this subsection adds the following assumption:

$$\frac{\partial^2 \Psi_U(n)}{\partial U \partial n} < 0. \tag{14}$$

For any given n, the indifference curve associated with higher U has the steeper (negative) slope than that associated with lower U. This assumption is not too restrictive: if preferences are

investment cost is a strict convex function of the measure of entrants or if the output produced in this economy faces a downward-sloping demand curve, then the multiplicity vanishes.

homothetic, (14) is satisfied. The assumption implies that

$$\frac{\partial^2 \Omega_U(l)}{\partial U \partial l} = \frac{1}{(1-t_w)\tau}\left[-(1+t_c)\frac{\partial^2 \Psi_U(n)}{\partial U \partial n}\right] > 0. \tag{15}$$

For any given l, the induced indifference curve associated with higher U has the steeper (positive) slope than that associated with lower U.

Changes in policy variables affect firms' and individuals' behavior in equilibrium through two channels. One is the *substitution effect*: changes in policy variables vary the form of the induced indifference curves in the (l, w) plane (i.e., the marginal rate of substitution between l and w). The other is the *wealth effect*: policy changes affect the firms' profits and their entry decision, resulting in changes in the utility level of individuals and thus changes in their behavior.

Lemma 4. *The wealth effect on fertility is positive: if $U < U'$, then $n_U(\alpha) < n_{U'}(\alpha)$ for all α.*

Proof. Totally differentiating (8) and using Lemma 1 and (15), we obtain

$$\frac{dl_U(\alpha)}{dU} = -\frac{\frac{\partial^2 \Omega_U(l)}{\partial U \partial l}}{\frac{\partial^2 \Omega_U(l)}{\partial l^2}} < 0.$$

This immediately leads us to the result. □

Equation (15) alone represents the wealth effect: the induced indifference curve associated with higher U has a steeper (positive) slope than that associated with lower U. Thus, jobs with productivity α decrease working hours (i.e., increase fertility) and increase compensation in response to an increase in U. The intuition is simple. For guaranteeing a higher level of utility, it is more profitable for jobs to increase a bit of both fertility and compensation rather than increase only one of these by a large margin. Thus, the wealth effect on fertility is positive. The effects of changes in policy variables on fertility choice are summarized below.

Proposition 3. *The effects of policy changes on fertility of employees in firms with productivity α are as follows:*

(a) *If t_c increases, fertility increases through the substitution effect but decreases through the wealth effect. The total effect is ambiguous.*

(b) *If t_w increases, fertility increases through the substitution effect but decreases through the wealth effect. The total effect is ambiguous.*

(c) *If s_n increases, fertility increases through both the substitution and wealth effects.*

(d) *If t_Π increases, fertility decreases through the wealth effect.*

Proof. (a) Totally differentiating (8), we obtain

$$\frac{\partial l_U(\alpha)}{\partial t_c} = \frac{\tau \frac{\partial \Psi_U(n)}{\partial n}}{(1+t_c)\frac{\partial^2 \Psi_U(n)}{\partial n^2}} < 0. \tag{16}$$

This implies $\partial n_U(\alpha)/\partial t_c > 0$, and thus fertility increases through the substitution effect. Differentiating (10) with respect to t_c and using the envelope theorem, we obtain

$$\frac{\partial \Pi_U^*(\alpha)}{\partial t_c} = -\frac{1}{1-t_w}\Psi_U\left(\frac{1-l}{\tau}\right) < 0. \tag{17}$$

Since an increase in t_c decreases the profit for all α, the expected profit of entry falls and the measure of entrants decreases in response to an increase in t_c. The utility level must fall to restore the balance of supply and demand in the labor market. It follows from Lemma 4 that fertility decreases through the wealth effect. (b), (c), and (d): In the same way, we establish that $\partial l_U(\alpha)/\partial t_w < 0$, $\partial \Pi_U^*(\alpha)/\partial t_w < 0$, $\partial l_U(\alpha)/\partial s_n < 0$, $\partial \Pi_U^*(\alpha)/\partial s_n > 0$, $\partial l_U(\alpha)/\partial t_\Pi = 0$, and $\partial \Pi_U^*(\alpha)/\partial t_\Pi < 0$. □

We can understand the substitution effect by investigating how the first-order condition, (8), varies. The first-order condition represents the allocation when U is exogenously given. Thus, (16) indicates that if the utility level remains unchanged, an increase in consumption tax rate reduces working hours (i.e., increases child-rearing time) in jobs with productivity α. The slope of the induced indifference curve is the increment of wages required to induce individuals to work an extra hour. An increase in consumption tax rate makes the required increment larger, and as a result, firms become reluctant to offer labor contracts with longer working hours (i.e., shorter child-rearing time). This acts to increase the number of children. That is not the only effect, however, because policy changes also affect the utility level of individuals. We can understand the wealth effect by investigating how the firms' profit, (10), varies. If the expected profit of entry decreases as a result of policy changes, then the measure of entrants decreases. Since the excess supply of labor arises, the utility level declines, the threshold level of productivity falls, and then, the balance of supply and demand is restored. Thus, (17) implies that an increase in consumption tax rate lowers the profit, curbing the jobs' entry and decreasing the utility level. Since the wealth effect on fertility is positive, a decrease in U lowers fertility. It is ambiguous whether employees in jobs with productivity α increase or decrease the number of children in response to an increase in t_c because the substitution and wealth effects have opposing effects on fertility. It follows from the analogous arguments that we can understand the effects of changes in other policy variables: the effects of increases in t_w, s_n, and t_Π on fertility choice are ambiguous, positive, and negative, respectively.

The focus of the above analysis is on the fertility choice of each employee. The effects of policy changes on the aggregate fertility are more complex. The average fertility rate of this economy is

$$\overline{n^*} \equiv \frac{1}{1-F\left(\underline{\alpha}^*\right)} \int_{\underline{\alpha}^*}^{\infty} n^*\left(\alpha\right) dF\left(\alpha\right).$$

To examine the effects of policy changes on $\overline{n^*}$, we must consider not only the behavior of each agent but also the *composition effect* because the threshold level $\underline{\alpha}^*$ is endogenous and policy changes might affect the productivity distribution among active jobs by affecting $\underline{\alpha}^*$. For instance, the effect of an increase in consumption tax rate on $\overline{n}^*$ is

$$\frac{\partial \overline{n}^*}{\partial t_c} = \left[\frac{\int_{\underline{\alpha}^*}^{\infty} n^* (\alpha) \, dF (\alpha)}{1 - F (\underline{\alpha}^*)} - n^* (\underline{\alpha}^*) \right] f (\underline{\alpha}^*) \left[1 - F (\underline{\alpha}^*) \right] \frac{\partial \underline{\alpha}^*}{\partial t_c} + \int_{\underline{\alpha}^*}^{\infty} \frac{\partial n^* (\alpha)}{\partial t_c} f (\alpha) \, d\alpha, \tag{18}$$

where $f(\alpha)$ is the density. The first and second terms represent the composition effect and the effect discussed in Proposition 3, respectively.[12] Since individuals in higher-productivity jobs have fewer children, $\int_{\underline{\alpha}^*}^{\infty} n^* (\alpha) \, dF (\alpha) / [1 - F (\underline{\alpha}^*)] < n^* (\underline{\alpha}^*)$. Thus, the sign of the composition effect depends on the sign of $\partial \underline{\alpha}^* / \partial t_c$: through the composition effect, increase (resp. decrease) in the threshold productivity decreases (resp. increases) the average fertility rate.

Let us elaborate the composition effect. In the competitive equilibrium where there are positive entrants, the following two conditions must hold: (i) the threshold job's profit is 0 (i.e., $\Pi^* (\underline{\alpha}^*) = 0$) and (ii) the expected profit at the time of entry is 0 (i.e., $E (\Pi^* (\alpha)) - \kappa = 0$). Investigating these two conditions leads us to the following proposition.

Proposition 4. *The effects of policy changes on the exit threshold $\underline{\alpha}$ are as follows:*

(a) If the utility elasticity of consumption increases with fertility (i.e., $\partial \left[(\partial \Psi_U / \partial U) (U / \Psi_U) \right] / \partial n > 0$), then an increase in t_c decreases the threshold, and vice versa.

(b) It is ambiguous whether an increase in t_w increases or decreases the threshold.

(c) An increase in s_n decreases the threshold.

(d) An increase in t_Π decreases the threshold.

Proof. See Appendix. □

[12] Note that $\partial n^* (\alpha) / \partial t_c$ differs from $\partial n_U^* (\alpha) / \partial t_c$: $\partial n_U^* (\alpha) / \partial t_c$ is just the substitution effect (i.e., the effect when keeping U constant), while $\partial n^* (\alpha) / \partial t_c$ includes the wealth effect as well as the substitution effect.

As shown in Proposition 3, when t_c increases, the equilibrium is restored by a reduction in U. Since an increase in t_c directly curbs the entry whereas a reduction in U promotes the entry, whether the measure of entrants in the new equilibrium is larger or smaller than the measure of entrants in the initial equilibrium depends on which effect is dominant. Proposition 4 states that the effect of an increase in t_c depends on the form of preferences, the effect of increase in t_w depends not only on the form of preferences but also on other parameters, and the effects of increases in s_n and t_Π on the threshold are negative.

Combining the results of Propositions 3 and 4 enables us to understand the effects of policy changes on the average fertility rate: Propositions 3 and 4 respectively indicate the signs of the second and first terms in (18). It is only for the child-rearing subsidy, s_n, that whether its change raises the average fertility rate or not is unambiguously determined: increase in s_n necessarily raises the average fertility rate.

Next, we examine the effect of technological progress. Although various formulations might be possible as a representation of technological progress, here, we capture the technological progress as a change in the productivity distribution from $F(\cdot)$ to $G(\cdot)$ such that $G(\cdot)$ first-order stochastically dominates $F(\cdot)$, i.e., $F(\alpha) \geq G(\alpha)$ for all α.

Proposition 5. *A change in the productivity distribution from $F(\cdot)$ to $G(\cdot)$, where $G(\cdot)$ first-order stochastically dominates $F(\cdot)$, raises the average fertility through the wealth effect but lowers it through the composition effect. The total effect is ambiguous.*

Proof. Since $\Pi^*(\alpha)$ is nondecreasing function, it follows from the first-order stochastic dominance that the following inequality holds:

$$\int_{\underline{\alpha}^*}^{\infty} \Pi^*(\alpha)\, dG(\alpha) \geq \int_{\underline{\alpha}^*}^{\infty} \Pi^*(\alpha)\, dF(\alpha) = \kappa.$$

Thus, such a change in the productivity distribution promotes entry of jobs, increases the exit threshold, and raises the utility level. By the wealth effect, employees in jobs with productivity α increase children for all α. By the composition effect, on the other hand, the proportion of employees in higher-productivity jobs, who have fewer children, increases. It is ambiguous whether average fertility rate increases or decreases because the wealth and composition effects act in opposite directions. □

Cross-sectionally, fertility falls as wages rise. However, an increase in the overall wage level caused by technological progress does not necessarily result in a fall in the overall fertility rate, because of the wealth effect: an increase in the utility level triggered by technological progress induces jobs with productivity α to offer shorter working hours (i.e., higher fertility) for all α. If the wealth effect dominates the composition effect, technological progress leads to an increase in the average fertility rate, and vice versa. This property enables our model to reconcile the negative cross-sectional wage-fertility relationship with various time-series variation in the aggregate fertility over the course of economic growth and business cycle.

The effect of the technological progress considered here is similar to the effect of a fall in the corporate-profit tax rate. Both have no substitution effect and have the positive wealth and negative composition effects on the average fertility rate. However, note that the effects of technological progress and a fall in the corporate-profit tax rate on the average fertility rate are not identical. A change in the corporate-profit tax rate affects the average fertility rate only by affecting the measure of entrants. On the other hand, a change in the productivity distribution varies the average fertility rate even if it does not entail a change in the measure of entrants. Suppose that a change in the expected profit is offset by a proportionate change in the initial investment cost and the measure of entrants does not change. In this case, a change in the corporate-profit tax rate has no effect on the equilibrium allocation of this economy. On the other hand, a change in the productivity distribution varies the average fertility rate. If the equilibrium measures of entrants under $F(\cdot)$ and $G(\cdot)$ are the same, then the following must hold for the balance of supply and demand in the labor market:

$$F(\underline{\alpha}) = G(\underline{\alpha}'),$$

where $\underline{\alpha}$ and $\underline{\alpha}'$ are the equilibrium exit thresholds under $F(\cdot)$ and $G(\cdot)$, respectively. It follows from the first-order stochastic dominance that $\underline{\alpha} < \underline{\alpha}'$. The higher threshold level is associated with the higher utility level. Even if technological progress is accompanied by an increase in the entry cost and the resultant measure of entrants does not change, the positive wealth and negative composition effects coexist, and it is ambiguous whether the resultant average fertility rate increases or not.

3 Example: Constant wage elasticity of fertility

Thus far, we have tried to keep the model as general as possible to illustrate the robustness of our theory. Since one broad aim of this paper is to offer an "off-the-shelf" fertility model as a building block in applied research, this section shows that specifying functional forms enables our model to derive a closed-form solution, illustrating the tractability of our theory. This example predicts a constant wage elasticity of fertility, which is consistent with a finding reported in Jones and Tertilt (2008). Based on the parameterized model, furthermore, we examine the effects of technological change, illustrating that our theory can reconcile the negative cross-sectional wage-fertility relationship with the various time-series variation in aggregate fertility over the course of economic growth and business cycle.

To parameterize our model, we need to specify the utility function and the productivity distribution. We assume a Cobb-Douglas utility function, which is commonly used in the literature:

$$u(n,c) = n^{\gamma} c^{1-\gamma},$$

where $\gamma \in (0,1)$ represents the relative weight for children. Assume that the productivity distribution, $F(\alpha)$, is a uniform distribution over $[0,A]$:

$$F(\alpha) = \begin{cases} \frac{\alpha}{A} & \text{if } \alpha \in [0,A], \\ \\ 1 & \text{if } \alpha > A. \end{cases}$$

The parameter $A > 0$ represents the maximum technology level available in this economy. Here, technological progress is captured by an increase in A (the distribution with higher A first-order stochastically dominates the one with lower A). We assume that increase in A might be also associated with increase in costs: the jobs' initial investment costs and the individuals' per-child time costs are respectively given by $\kappa(A)$ and $\tau(A)$. This formulation allows us to include the case that the required investment and education costs increase with the rise of technology. Especially, the initial investment cost has to depend on A for balanced growth.

For simplification, we ignore tax, subsidy, and non-labor income here. Then, the time constraint and the budget constraint are, respectively, $\tau(A)n + l = 1$ and $c = w$. Before proceeding, we would like to emphasize that based on the conventional fertility model, where individuals facing given per-time wages allocate their time between labor supply and child rearing, we cannot derive the negative relationship between wages and fertility under the assumption that preferences are given by a Cobb-Douglas utility function and child rearing requires only the time costs: since the wealth and substitution effects cancel out each other, fertility does not vary with wages. As will become apparent, in contrast, this model produces the negative relationship.

A firm with productivity α solves the following problem:

$$\max_{l\in[0,1],w\geq 0} \alpha l - w$$

$$\text{s.t. } n^{\gamma}c^{1-\gamma} = U,\ \tau(A)n + l = 1, \text{ and } c = w$$

The first-order conditions imply

$$l_U(\alpha) = 1 - \left[\frac{(1-\gamma)\,\alpha}{\gamma}\right]^{\gamma-1} [\tau(A)]^{\gamma} U,$$

$$n_U(\alpha) = \left[\frac{(1-\gamma)\,\alpha\tau(A)}{\gamma}\right]^{\gamma-1} U, \tag{19}$$

and

$$w_U(\alpha) = \left[\frac{(1-\gamma)\,\alpha\tau(A)}{\gamma}\right]^{\gamma} U. \tag{20}$$

These establish that higher-productivity vacancies offer labor contracts with longer working hours (i.e., lower fertility) and higher wages. It should also be noted that an increase in the utility level, U, increases both $n_U(\alpha)$ and $w_U(\alpha)$. As shown above, the wealth effects on fertility and wages are positive (the utility function assumed here satisfies (14)). Using (19) and (20), we obtain

$$n_U = U^{\frac{1}{\gamma}} w_U^{-\frac{1-\gamma}{\gamma}},$$

which indicates the negative wage-fertility relationship: the wage elasticity of fertility is $-(1-\gamma)/\gamma$.

The maximized profit of a job with productivity α is

$$\Pi_U^*(\alpha) = \alpha - \frac{[\alpha\tau(A)]^{\gamma} U}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}. \tag{21}$$

It follows from $\Pi_U^*(\underline{\alpha}_U) = 0$ that the threshold productivity is

$$\underline{\alpha}_U = \left\{\frac{[\tau(A)]^{\gamma} U}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right\}^{\frac{1}{1-\gamma}}. \tag{22}$$

Suppose that $U > U_0$. Then, the market-clearing condition of the labor market, $N = [1 - F(\underline{\alpha}_U)]M$, is

$$N = \left\{1 - \frac{1}{A}\left[\frac{[\tau(A)]^{\gamma} U}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}\right\} M.$$

Using this, we can write the individuals' utility level as a function of the measure of entrants:

$$U = \frac{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}{[\tau(A)]^{\gamma}} \left(A \frac{M-N}{M} \right)^{1-\gamma}. \tag{23}$$

It follows from (21) that the expected profit of entry is

$$\int_{\underline{\alpha}_U}^{A} \Pi_U^{*}(\alpha)\, dF(\alpha) = \int_{\underline{\alpha}_U}^{A} \left\{ \alpha - \frac{[\alpha \tau(A)]^{\gamma} U}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right\} dF(\alpha)$$

Using (22) and (23), we can write the expected profit of entry as a function of the measure of entrants:

$$\int_{\underline{\alpha}_{U(M)}}^{A} \Pi_{U(M)}^{*}(\alpha)\, dF(\alpha) = A \left[\frac{1+\gamma-2\left(\frac{M-N}{M}\right)^{1-\gamma} + (1-\gamma)\left(\frac{M-N}{M}\right)^{2}}{2(1+\gamma)} \right]$$

Equalizing this to the initial investment cost, $\kappa(A)$, we obtain the free-entry condition:

$$A \frac{1+\gamma-2\left(\frac{M-N}{M}\right)^{1-\gamma} + (1-\gamma)\left(\frac{M-N}{M}\right)^{2}}{2(1+\gamma)} = \kappa(A). \tag{24}$$

Whether an increase in A promotes entry or not depends on the form of $\kappa(\cdot)$. If $\kappa(A)/A$ decreases (resp. increases) with A, then the measure of entrants increases (resp. decreases) with A.

Now all the equilibrium conditions are set. Using them, we can compute the average fertility rate:

$$\bar{n} = \frac{1}{1-F(\underline{a}_U)} \int_{\underline{\alpha}_U}^{A} n(\alpha)\, dF(\alpha) = \frac{1}{1-\frac{M-N}{M}} \frac{\left(\frac{M-N}{M}\right)^{1-\gamma} - \frac{M-N}{M}}{\tau(A)}.$$

A change in A affects the average fertility rate through two channels: one directly affects child-rearing costs, $\tau(A)$, and the other affects the measure of entrants (i.e., through equation (24)).

We conclude this section by presenting a numerical example. We consider three cases: (i) $\kappa(A) = \kappa$ and $\tau(A) = \tau$, (ii) $\kappa(A) = A\kappa$ and $\tau(A) = \tau$, and (iii) $\kappa(A) = A\kappa$ and $\tau(A) = A\tau$. The

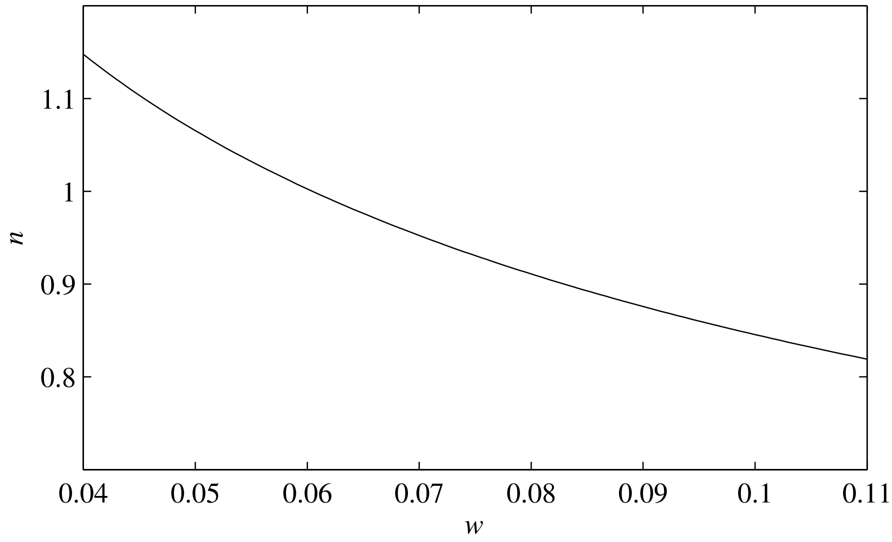


Figure 4: Wage-fertility relationship.

measure of individuals, N, is normalized to 1. We assume that $\gamma = 0.75$ (the relative weight for children in the utility function),[13] $\kappa = 0.25$, $\tau = 0.4$, and $A = 1$. It follows from the free-entry condition, (24), that the associated measure of entrants, utility level, and threshold productivity are, respectively, $M^* = 1.038$, $U^* = 0.496$, and $\underline{\alpha}^* = 0.301$. The lowest and highest wages are $w(\underline{\alpha}^*) = 0.045$ and $w(1) = 0.109$, respectively. Figure 4 depicts the wage-fertility relationship. Figure 5 exhibits how the average fertility rate changes as A changes in the cases of (i), (ii), and (iii): in case (i), increase in A entail the dominant wealth effect, raising the aggregate fertility (the upward dotted line); in case (ii), the wealth and composition effects associated with increase in A cancel out each other, and thus, the aggregate fertility does not change (the horizontal dotted line); in case (iii), in addition to the effects observed in the case (ii), increase in child-rearing costs directly decrease the aggregate fertility (the downward dotted line). The cross-sectional wage-fertility relationship is negative at any point in Figure 5. Our model reconciles the negative cross-sectional wage-fertility relationship with various time-series variation in the

[13] This value of γ corresponds to the wage elasticity of fertility of $-1/3$. Jones and Tertilt (2008) find that for 30 birth cohorts between 1830 and 1960 in the United States, the income elasticity of fertility remained roughly constant at about -0.30.

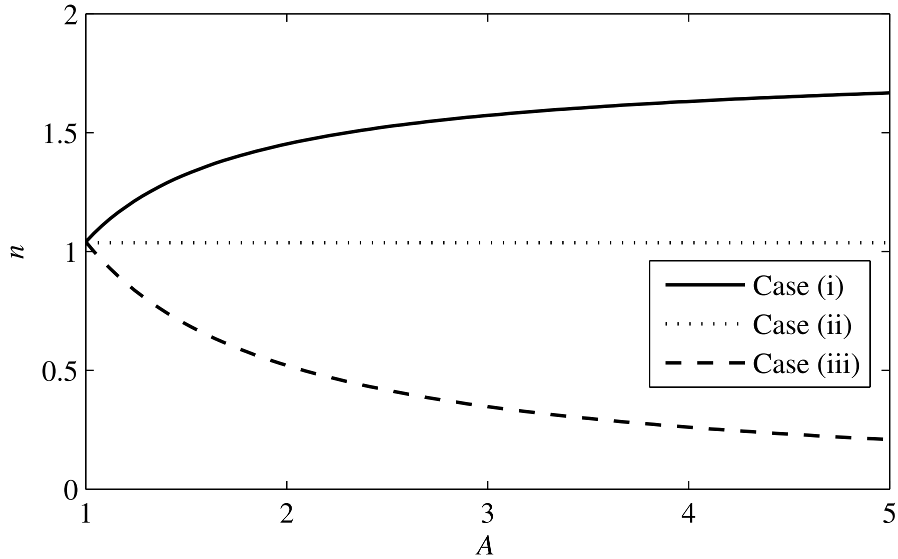


Figure 5: Changes in average fertility in response to changes in A. Case (i): upward line. Case (ii): horizontal dotted line. Case (iii): downward dotted line.

average fertility rate.

4 Extension

4.1 Quantity-quality tradeoff

It is rather standard in the literature to include in parents' preferences not only the number of children, but also some measure of the quality of children. In this subsection, we extend the model so that parents care about the educational attainment of their children, and we explore the robustness of our model to such an extension. The utility function is modified as $u(n,c,h)$, where h is the educational attainment of children. As in the basic model, the utility function is strictly increasing, strictly quasi-concave, and twice continuously differentiable and satisfies an Inada-type condition. We assume that child rearing takes time while education requires purchased inputs as in de la Croix and Doepke (2003) and Moav (2005).[14]

[14]Although this assumption is often used in the literature, we do not need it to derive the negative wage-fertility relationship. However, with this assumption, we can derive the positive relationship between parental wages and

In this subsection, we introduce the leisure time into the model, despite not generating utility. Producing a child not only takes time but also increases education costs, which are the essence of the quantity-quality model that the product of quality and quantity of children appears in the budget constraint (see Equation (25)). It might be preferable for individuals to discard a part of the time available for reducing education expenditure, then they might wish to work longer hours without increases in compensation. To avoid such an implausible situation, we introduce the disposable time as "leisure" and denote it by z. The time constraint, the budget constraint, and the education function are respectively given by

$$\tau n + l + z = 1,$$

$$(1+t_c)\,c + en = (1-t_w)\,w + \theta + s_n n, \tag{25}$$

and

$$h = m(e),$$

where e is the education spending per child and $m(\cdot)$ is the education production function. We assume that $m(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable.

An important departure from the basic model is that once offered labor contract (l, w), individuals further allocate their time between child rearing, τn, and leisure, z, and allocate their income between education for children, en, and own consumption, c, to maximize their utility. Given this individual maximization behavior, each firm chooses a contract to post. To solve the model backward, we first consider the individuals' problem. Given l and w, an individual solves the following maximization problem:

$$\max\ u(n, c, h),$$

$$\text{s.t. } \tau n + l + z = 1,\ (1+t_c)\,c + en = (1-t_w)\,w + \theta + s_n n, \text{ and } h = m(e).$$

educational level, which is widely observed in the reality, under moderate restrictions on preferences as well.

The first-order conditions of this problem are given by

$$-u_1 - u_2 \frac{s_n - e}{1+t_c} \leq 0, \tag{26}$$

with equality if $z > 0$, and

$$u_2 \frac{1}{1+t_c} \frac{1-l-z}{\tau} = u_3 m'(e), \tag{27}$$

where u_i represents the partial derivative of u with respect to the i-th argument. It follows from strict quasi concavity of u that the second-order condition is satisfied.

Therefore, the maximization problem faced by a job with productivity α is the following:

$$\max_{l \in [0,1], w \geq 0} \Pi(\alpha) = (1 - t_\Pi)(\alpha l - w),$$

$$\text{s.t. } u\left(\frac{1-l-\tilde{z}}{\tau}, \frac{(1-t_w)w + \theta + (s_n - \tilde{e})\frac{1-l-\tilde{z}}{\tau}}{1+t_c}, m(\tilde{e})\right) = U, \tag{28}$$

where $\tilde{z}$ and $\tilde{e}$ are given by (26) and (27). Solving this problem leads us to the following proposition.

Proposition 6. *(a) A job with higher productivity offers a labor contract with longer working hours and higher compensation. An employee in a higher-productivity job has fewer children. (b) An employee in a higher-productivity job educates children more if and only if*

$$(u_{22}u_3 - u_{32}u_2)u_1 + (u_{31}u_2 - u_3u_{21})u_2 < 0. \tag{29}$$

Proof. (a) Showing that induced indifference curves in the (l, w) plane, (28), are upward sloping and convex to the l axis proves part (a) of this proposition. Totally differentiating (28), we obtain

$$\frac{dw}{dl} = \frac{1}{\tau} \frac{1+t_c}{1-t_w} \left(\frac{u_1}{u_2} + \frac{s_n - \tilde{e}}{1+t_c}\right) \geq 0. \tag{30}$$

This sign comes from (26). Consider the case where (30) holds with strict inequality.[15] Differentiating (30) with respect to l and noting that the relationship between l and w on an induced indifference curve is given by (30), we obtain

$$\frac{d^2w}{dl^2} = -\frac{1+t_c}{(1-t_w)\,\tau^2}\frac{u_2^2u_{11}-2u_1u_2u_{12}+u_1^2u_{22}}{u_2^3} > 0.$$

This sign comes from strict quasi concavity of u. Induced indifference curves in the (l,w) plane are upward sloping and convex to the l axis, as in the basic model (Figure 1). Thus, part (a) of this proposition follows from the analogous argument of Proposition 1. (b) Totally differentiating (27) with respect to w, l, and e, and using (27) and (30), we obtain

$$\begin{aligned}&\frac{(u_{22}u_3-u_{32}u_2)\,u_1+(u_{31}u_2-u_3u_{21})\,u_2}{u_2u_3}\frac{1}{\tau}\frac{n}{1+t_c}dl\\ &=\left[\left(u_{22}u_3^2-2u_{23}u_3u_2+u_{33}u_2^2\right)\frac{1}{u_3^2}\frac{n^2}{(1+t_c)^2}+u_3m''(e)\right]de.\end{aligned}$$

It follows from strict quasi concavity of u that $u_{22}u_3^2-2u_{23}u_3u_2+u_{33}u_2^2<0$. Therefore, $de/dl<0$ if and only if the inequality (29) holds. □

Even if the quantity-quality tradeoff is introduced, the model can generate the negative wage-fertility relationship using only standard conditions on preferences (quasi concavity) and the quality-production function (concavity). A widespread finding in the literature is that wealthier parents tend to have fewer children, as well as educate them more. To explain this finding based on our model, we need an additional assumption on preferences because the strict quasi concavity of u alone does not guarantee (29). The assumption of homothetic preferences qualifies as the additional assumption required. Adding the homotheticity of u establishes that $(u_{22}u_3-u_{32}u_2)\,u_1+(u_{31}u_2-u_3u_{21})\,u_2=(u_{22}u_3-u_{32}u_2)\,u_1<0$. The assumptions required here to obtain the desired result are still considerably moderate compared to most of the existing lit-

[15] There might be regions where $dw/dl=0$ (i.e., $z>0$). However, points (l,w) belonging to such regions are not offered because they are inconsistent with firms' profit maximization: if firms do not need to raise wages, they want their employees to work as long as possible. Thus, we can focus on the case where $dw/dl>0$.

erature.

4.2 Nannies

What happens in our model if parents are allowed to outsource childcare? Although it is fairly obvious that childcare is a time-intensive activity, childcare can be, at least partly, bought in the market (e.g., nannies, nurseries, and wet nurses) and outsourcing childcare has been growing in developed countries. In this subsection, we extend the model so that parents are allowed to outsource childcare, and we explore the robustness of our model to such an extension. The time constraint and the budget constraint are respectively given by

$$(1-q)\,\tau n + l = 1$$

and

$$(1+t_c)\,[c + v(q)\,q\tau n] = (1-t_w)\,w + \theta + s_n n, \tag{31}$$

where $q \in [0,1]$ is the proportion of the nanny's time and $v(q)$ is the cost of a nanny per unit of time. It is assumed that $v(\cdot)$ is strictly increasing, twice continuously differentiable, and $\lim_{q\to 1} v(q) = \infty$. Furthermore, we assume that

$$v''(q)\,q + 2v'(q) > 0, \tag{32}$$

which is the condition for $\partial^2 [v(q)\,q\tau n] / \partial q^2 > 0$ to be satisfied. These assumptions on $v(\cdot)$ capture the idea that there are various activities in childcare, they differ in cost, and individuals outsource childcare in ascending order of cost.

Given an offered labor contract (l, w), individuals further allocate their income between the nanny's cost, $v(q)\,q\tau n$, and own consumption, c, to maximize their utility. Given this individual maximization behavior, each firm chooses a contract to post. To solve the model backward, we first consider the individuals' problem. Given l and w, an individual solves the following maximization problem:

$$\max\ u(n,c),$$

$$\text{s.t. } (1-q)\,\tau n + l = 1 \text{ and } (1+t_c)\,[c + v(q)\,q\tau n] = (1-t_w)\,w + \theta + s_n n.$$

The first-order condition of this problem is given by

$$\frac{u_1}{u_2} + \frac{s_n}{1+t_c} - \left[v'(q)\,q\,(1-q) + v(q)\right]\tau \le 0, \tag{33}$$

with equality if $q > 0$. Note that $q = 0$ if $u_1/u_2 + s_n/(1+t_c) - v(0)\,\tau \le 0$ and that the case of $q = 1$ is excluded by the assumption of $\lim_{q\to 1} v(q) = \infty$.

Therefore, the maximization problem faced by a job with productivity α is the following:

$$\max_{l\in[0,1],w\ge 0}\ \Pi(\alpha) = (1-t_\Pi)\,(\alpha l - w),$$

$$\text{s.t. } u\left(\frac{1-l}{(1-\tilde{q})\,\tau}, \frac{(1-t_w)\,w + \theta + s_n\frac{1-l}{(1-\tilde{q})\tau}}{1+t_c} - v(\tilde{q})\,\tilde{q}\frac{1-l}{(1-\tilde{q})}\right) = U, \tag{34}$$

where $\tilde{q}$ is given by (33). Solving this problem leads us to the following proposition.

Proposition 7. *(a) A job with higher productivity offers a labor contract with longer working hours and higher compensation. (b) An employee in a higher-productivity job has fewer children.*

Proof. (a) Showing that induced indifference curves in the (l,w) plane, (34), are upward sloping and convex to the l axis proves the part (a) of this proposition. Totally differentiating (34), we obtain

$$\frac{dw}{dl} = \frac{1+t_c}{(1-t_w)\,(1-\tilde{q})\,\tau}\left[\frac{u_1}{u_2} + \frac{s_n}{1+t_c} - v(\tilde{q})\,\tilde{q}\tau\right] > 0. \tag{35}$$

This sign obviously holds in the case of $\tilde{q} = 0$. In the case of $\tilde{q} > 0$, the sign holds because (33) holds with equality. Differentiating (35) with respect to l and noting that the relationship between l and w on an induced indifference curve is given by (35), we obtain

$$\frac{d^2w}{dl^2} = -\frac{1+t_c}{(1-t_w)(1-\tilde{q})^2\tau^2}\frac{u_2^2u_{11}-2u_1u_2u_{12}+u_1^2u_{22}}{u_2^3} > 0.$$

This sign comes from strict quasi concavity of u. Induced indifference curves in the (l,w) plane are upward sloping and convex to the l axis, as in the basic model (Figure 1). Thus, the part (a) of this proposition follows from the analogous argument of Proposition 1. (b) Since childcare can be outsourced, longer working hours do not necessarily mean fewer children. The number of children is given by $n = (1-l)/[(1-q)\tau]$. Thus, noting that the value of q chosen by an employee depends on l, the relationship between working hours and fertility is given by

$$\frac{\partial n}{\partial l} = \frac{1}{\tau}\frac{-(1-q)+(1-l)\frac{dq}{dl}}{(1-q)^2}. \tag{36}$$

Totally differentiating (33), we obtain

$$\frac{dq}{dl} = \frac{\frac{1}{(1-q)\tau}\frac{u_2^2u_{11}-2u_1u_2u_{12}+u_1^2u_{22}}{u_2^3}}{\frac{1-l}{(1-q)^2\tau}\frac{u_2^2u_{11}-2u_1u_2u_{12}+u_1^2u_{22}}{u_2^3}-[v''(q)q+2v'(q)](1-q)\tau}.$$

Substituting this into (36), we obtain

$$\frac{\partial n}{\partial l} = \frac{[v''(q)q+2v'(q)]\frac{(1-q)^2\tau}{(1-l)}}{\frac{u_2^2u_{11}-2u_1u_2u_{12}+u_1^2u_{22}}{u_2^3}-[v''(q)q+2v'(q)]\frac{(1-q)^3\tau^2}{(1-l)}} < 0. \tag{37}$$

This sign comes from (32) and strict quasi concavity of u. □

Even if parents are allowed to outsource childcare, the model can generate the negative wage-fertility relationship. The intuition is simple. Owing to the assumption that the cost of a nanny per unit of time increases with the proportion of the nanny's time, individuals with longer working hours (i.e., shorter own childcare time) face a higher price of childcare. Thus, they choose lower fertility and higher consumption to obtain the same utility level as individuals with shorter working hours obtain. Equation (37) indicates that though the negative wage-fertility relationship is still preserved, the improvement in the availability of market childcare reduces the fertility differential. Let us interpret the size of $v''(q)q + 2v'(q)$ as the difficulty in using market childcare. The absolute value of $\partial n/\partial l$ decreases as $v''(q)q + 2v'(q)$ decreases and converges to 0 as $v''(q)q + 2v'(q)$ approaches 0. The linear cost (i.e., $v(q)$ is constant) means that $v''(q)q + 2v'(q) = 0$; then, $\partial n/\partial l < 0$ for $q = 0$, while $\partial n/\partial l = 0$ for $q > 0$. This implies that if all types of childcare service can be purchased at the same price, individuals who use nannies' time have the same number of children: the difference in own childcare time is offset by the use of market childcare.

5 Discussion

5.1 Jobs' heterogeneity vs individuals' heterogeneity

This subsection organizes the relationship between the jobs' heterogeneity approach (ours) and the individuals' heterogeneity approach (the conventional one). We introduce individuals' ability heterogeneity into our framework, illustrating that in contrast with the negative wage-fertility relationship stemming from the jobs' heterogeneity, the one stemming from the individuals' heterogeneity depends on the form of preferences. It establishes that the robustness of our theory comes not from the model's structure (wages and fertility are simultaneously determined in a contract-posting market) but from jobs' heterogeneity. The discussion here proposes an important viewpoint for the empirical literature.

For simplicity, suppose that there is no heterogeneity in jobs' productivity. There are two types in individuals' ability, β_1 and β_2 ($> \beta_1$). When a job hires an individual with ability β_i, the match produces β_i units of good per unit time. Individuals with different ability might have different utility levels, while homogeneous jobs must earn the same profit. As in the case where there is heterogeneity in jobs' productivity, we can graphically depict the offered contracts.

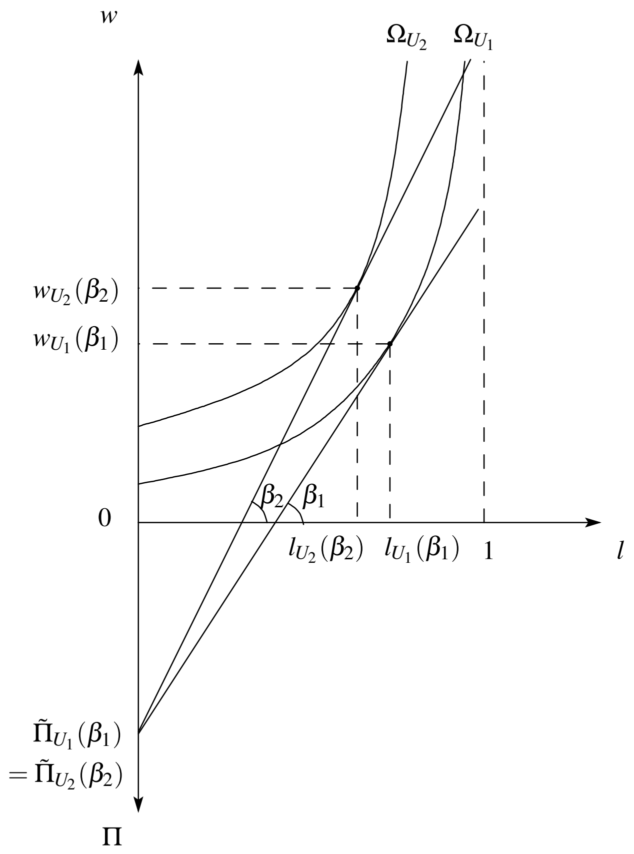


Figure 6: Labor contracts offered to individuals with β_1 and β_2 ($> \beta_1$).

Figure 6 exhibits a possible case. Since both the substitution and wealth effects on consumption (i.e., wages) are positive, a higher-ability individual receives higher wages. On the other hand, the substitution effect on fertility is negative while the wealth effect on fertility is positive; thus, it is ambiguous whether a higher-ability individual has more or less children. Figure 6 depicts the case where the wealth effect is dominant, and thus, a higher-ability individual has more children ($l_{U_1}(\beta_1) > l_{U_2}(\beta_2)$ implies $n_{U_1}(\beta_1) < n_{U_2}(\beta_2)$). This illustrates that the conventional fertility theory based on individuals' heterogeneity must impose restrictive assumptions on preferences to explain the negative relationship between wages and fertility. Furthermore, the wage-fertility relationship is not robust to changes in economic environments (e.g., policy changes) because they generally vary the form of the induced indifference curves. Even if the negative relationship holds under a parameter configuration, it does not necessarily hold under others.

The above description suggests that the cross-sectional wage-fertility relationship depends on the source of wage differential. Several studies in the literature argue that in the reality, the wealth effect is typically dominant and that the negative wage-fertility relationship is mainly a statistical fluke due to a problem of missing variables (e.g., Becker (1960), Becker and Lewis (1973), and Becker and Tomes (1976)). The argument here introduces a new insight into this strand of the literature: the source of wage differential, whether jobs' or individuals' heterogeneity, is a candidate for one of the variables missed in the past empirical studies.[16] The literature has often pointed out the importance of distinguishing sources of family income, for example, labor and non-labor income and husband and wife income.[17] The discussion here suggests that in addition to these, the source of wage differential also matters. Attempting to produce the negative wage-fertility relationship based solely on individuals' ability might be misleading and a careful examination on whether fertility differential stems from jobs' heterogeneity or individuals' heterogeneity is needed for calibrating and estimating the relevant parameters.

5.2 Exceptional findings

The negative relationship between wages and fertility is widespread across time and regions, but not universal. Several studies have reported exceptional findings, and one may argue that the robustness of our theory is inconsistent with these exceptions. However, it is inappropriate as a criticism toward the jobs' heterogeneity approach. The reality might be a hybrid of jobs' heterogeneity and individuals' heterogeneity. The jobs' heterogeneity approach is more likely to complement rather than contradict the individuals' heterogeneity approach. Combining both approaches might open the way for solving seemingly puzzling problems related to fertility

[16] There are a large number of empirical studies on fertility choice. For instance, they are surveyed in Hotz, Klerman and Willis (1997) and Schultz (1997).

[17] It is sometimes argued that an increase in family income due to an increase in the return on nonhuman assets is likely to raise fertility through the wealth effect, while that by an increase in wages is likely to reduce fertility through the substitution effect (e.g., Schultz (1981, 1994)), and an increase in the husband's wages is likely to raise fertility through the wealth effect, while an increase in the wife's wages is likely to reduce fertility through the substitution effect (e.g., Schultz (1986) and Blau and van der Klaauw (2007)).

choice. In what follows, we discuss two exceptional findings well known in the literature by combining both approaches.

Positive relationship in the early stage of development It is sometimes argued that in the early stage of the development process, there exists a positive income-fertility relationship (e.g., Wrigley (1961), Weir (1995), Clark and Hamilton (2006), Clark (2007), and Boberg-Fazlic, Sharp and Weisdorf (2011)). For instance, the following might be possible explanations to this finding. Jobs' productivity differential is large in advanced economies, while it is not in less-advanced ones; thus, wage differential in advanced economies largely reflects the jobs' productivity heterogeneity, while that in less-advanced economies largely reflects the individuals' ability heterogeneity. If that is the reality and an increase in wages associated with individuals' ability has a dominant wealth effect on fertility, the wage-fertility relationships in advanced and less-advanced economies are reversed, negative and positive, respectively. Alternatively, in agrarian economies, the mechanism proposed in this paper may not work well simply because the free choice of employment is not guaranteed. If individuals face a limited occupational choice, they are forced to accept their immediate environments, say, the farming villages they were born into, as their workplaces. Then, the utility levels among individuals are not equalized, and thus the individuals' heterogeneity approach fits well with the analysis on such economies.

Positive relationship within the same occupation Empirical findings in some earlier literature such as Freedman (1963) and Simon (1969) report that the overall cross-sectional relationship between wages and fertility is negative, whereas within the same occupation, higher-income households tend to have more children. Combining the jobs' heterogeneity approach with the individuals' heterogeneity approach might provide a theory reconciling the overall negative relationship with the within-occupation positive relationship. In many countries, the freedom to choose own occupation is guaranteed, but the individuals' performance in the occupation depends on their ability. This leads us to the supposition that inter-occupation wage differential largely reflects productivity differential between occupations, not inherent in individuals, whereas within-occupation wage differential largely reflects ability differential between individuals. Then, if the wealth effect associated with the individuals' ability heterogeneity dominates the substitution effect, the observed pattern is obtained.

Needless to say, the hypotheses presented above are just theoretical possibilities and should be empirically tested in a rigorous way. What we would like to emphasize here, however, is that the difference between implications drawn from the jobs' and individuals' heterogeneity approaches can broaden our options available to analyze fertility choice. We hope that the discussion presented here facilitates the testing of theories on fertility choice.

6 Conclusion

This paper has constructed a tractable theory of the cross-sectional fertility differential. The theory sheds light on heterogeneity of the jobs' productivity, one that has largely been ignored in the literature. By using only standard conditions on preferences, we show that the negative wage-fertility relationship can be produced and it is quite robust to changes in economic environments (e.g., public policy and technology). Our theory reconciles the negative cross-sectional wage-fertility relationship with various time-series variations in aggregate fertility over the course of economic growth and business cycle.

This paper is not intended to say that existing theories based on individuals' heterogeneity are wrong because they are not robust to changes in assumptions. On the contrary, we believe that our theory based on jobs' heterogeneity complements the existing literature to improve our understanding of fertility choice. The reality might be a hybrid of jobs' heterogeneity and individuals' heterogeneity. Making explicit the difference between implications drawn from the jobs' and individuals' heterogeneity approaches adds an important viewpoint to the literature. For instance, this paper shows that the cross-sectional wage-fertility relationship depends on the source of wage differential. Without the investigation on the source of wage differential, estimating the form of preferences from observed wage and fertility differentials might be misleading.

Since our model is simple and tractable, it can be theoretically extended in several ways. Let us propose some potential directions for future research. Explicitly distinguishing between the time of husband and wife would be an interesting extension. It shows what types of labor contracts are chosen by males and females and what types of couples are formed in equilibrium. Furthermore, extending our model to dynamic models or incorporating our theory into overlapping-generations frameworks is useful to analyze the cross-sectional relationship and the time-series variation simultaneously.

Although this paper is motivated by empirical patterns in fertility and the main aim is to develop a fertility-choice theory producing a robust negative wage-fertility relationship, our framework can also be used to analyze labor-leisure choice. If we modify the proposed model so that individuals receive utility from leisure time, instead of children, it becomes the model for analyzing labor-leisure choice. Modern-growth and business-cycle theories accept the long-run stability of leisure per capita as a stylized fact (Ramey and Francis (2009) provide a thorough evidence). Our model reconciles the long-run stability of leisure per capita with the cross-sectional upward-sloping labor supply curve and the time-series upward trends in real wages. Furthermore, almost all the arguments concerning fertility choice presented in this paper, for example, different effects between firms' and individuals' heterogeneity, are applied to the discussion on labor-leisure choice.

7 Appendix

7.1 Proof of Proposition 4

(a) Suppose that M is fixed. Since M is fixed, the exit threshold must remain unchanged to keep the balance of supply and demand in the labor market. Thus, the profit of the threshold firm must remain 0. For this, the utility level U must decline to offset the negative effect of an increase in

t_c on the profit. Totally differentiating $\Pi_U^*(\underline{\alpha}_U) = 0$, we obtain

$$\frac{dU}{dt_c} = -\frac{\Psi_U(n(\underline{\alpha}_U))}{(1+t_c)\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}},$$

which represents the iso-profit condition of the threshold job. To keep the profit of the threshold job 0, one unit of increase in t_c must be accompanied by $\Psi_U(n(\underline{\alpha}_U)) / [(1+t_c)\,\partial \Psi_U(n(\underline{\alpha}_U)) / \partial U]$ units of decrease in U. It follows that given M, the variation in the profit of jobs with productivity α, $\Pi^*(\alpha)$, caused by such a simultaneous change of t_c and U is

$$\begin{aligned} d\Pi^*(\alpha) &= \frac{\partial \Pi^*(\alpha)}{\partial t_c} dt_c + \frac{\partial \Pi^*(\alpha)}{\partial U} dU \\ &= \frac{\partial \Pi^*(\alpha)}{\partial t_c} dt_c - \frac{\partial \Pi^*(\alpha)}{\partial U} \frac{\Psi_U(n(\underline{\alpha}_U))}{(1+t_c)\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}} dt_c \\ &= -\frac{1-t_\Pi}{1-t_w}\left[\varepsilon_U(n(\underline{\alpha}_U)) - \varepsilon_U(n(\alpha))\right] \frac{\Psi_U(n(\underline{\alpha}_U))}{\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}} \frac{\Psi_U(n(\alpha))}{U} dt_c. \end{aligned}$$

where $\varepsilon_U(n(\alpha)) \equiv [\partial \Psi_U(n(\alpha)) / \partial U] \cdot [U / \Psi_U(n(\alpha))]$ is the utility elasticity of consumption evaluated at $n(\alpha)$. This implies that the profit of jobs facing the higher (resp. lower) utility elasticity of consumption than that the threshold job faces increases (resp. decreases). If the utility elasticity of consumption increases with fertility (i.e., $\partial [(\partial \Psi_U / \partial U)(U/\Psi_U)] / \partial n > 0$), then the profits of all jobs with productivity $\alpha > \underline{\alpha}$ fall because $n(\alpha) < n(\underline{\alpha})$ for all $\alpha > \underline{\alpha}$. Then, the expected profit of entry for given M falls, the measure of entrants decreases, and the exit threshold declines. Conversely, if the utility elasticity of consumption decreases with fertility, then the exit threshold rises.

(b) We follow a procedure similar to the proof of (a). The iso-profit condition of the threshold job is

$$\frac{dU}{dt_w} = -\frac{w_U(\underline{\alpha}_U)}{(1-t_w)(1+t_c)\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}}.$$

Given M, the variation in $\Pi^*(\alpha)$ caused by a simultaneous change of t_w and U such that $\Pi^*(\underline{\alpha})$

remains 0 is

$$
\begin{aligned}
d\Pi^*(\alpha) &= \frac{\partial \Pi^*(\alpha)}{\partial t_w} dt_w + \frac{\partial \Pi^*(\alpha)}{\partial U} dU \\
&= \frac{\partial \Pi^*(\alpha)}{\partial t_w} dt_w - \frac{\partial \Pi^*(\alpha)}{\partial U} \frac{w_U(\underline{\alpha}_U)}{(1-t_w)(1+t_c)\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}} dt_w \\
&= -\frac{1-t_\Pi}{(1-t_w)^2} \left[\frac{w_U(\alpha)}{\frac{\partial \Psi_U(n(\alpha))}{\partial U}} - \frac{w_U(\underline{\alpha}_U)}{\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}} \right] \frac{\partial \Psi_U(n(\alpha))}{\partial U} dt_w
\end{aligned}
$$

Since $w_U(\alpha) > w_U(\underline{\alpha}_U)$ and $\partial \Psi_U(n(\alpha))/\partial U > \partial \Psi_U(n(\underline{\alpha}_U))/\partial U$ for all $\alpha > \underline{\alpha}_U$, it is ambiguous whether it is positive or negative without further specification. Since $w_U(\alpha)$ depends not only on preferences, but also on policy variables, in contrast to the proof of (a), we cannot determine the sign of $d\Pi^*(\alpha)$ only by imposing a restriction on preferences.

(c) We follow a procedure similar to the proof of (a). The iso-profit condition of the threshold job is

$$
\frac{dU}{ds_n} = \frac{n(\underline{\alpha}_U)}{(1+t_c)\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}}.
$$

Given M, the variation in $\Pi^*(\alpha)$ caused by a simultaneous change of s_n and U such that $\Pi^*(\underline{\alpha})$ remains 0 is

$$
\begin{aligned}
d\Pi^*(\alpha) &= \frac{\partial \Pi^*(\alpha)}{\partial s_n} ds_n + \frac{\partial \Pi^*(\alpha)}{\partial U} dU \\
&= \frac{\partial \Pi^*(\alpha)}{\partial t_c} ds_n + \frac{\partial \Pi^*(\alpha)}{\partial U} \frac{n(\underline{\alpha}_U)}{(1+t_c)\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}} ds_n \\
&= \frac{1-t_\Pi}{1-t_w} \left[\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U} n(\alpha) - \frac{\partial \Psi_U(n(\alpha))}{\partial U} n(\underline{\alpha}_U) \right] \frac{1}{\frac{\partial \Psi_U(n(\underline{\alpha}_U))}{\partial U}} ds_n
\end{aligned}
$$

This is negative for all $\alpha > \underline{\alpha}_U$ because $n(\alpha) < n(\underline{\alpha}_U)$ and $\partial \Psi_U(n(\underline{\alpha}_U))/\partial U <$ $\partial \Psi_U(n(\alpha))/\partial U$ for all $\alpha > \underline{\alpha}_U$.Thus, the expected profit of entry for given M falls, the measure of entrants decreases, and the exit threshold declines.

(d) Even if t_Π changes, the profit of the initial exit threshold remains 0 $((1-t_\Pi)[\underline{\alpha}^* l(\underline{\alpha}^*) - w(\underline{\alpha}^*)] = 0)$. On the other hand, an increase in t_Π lowers the expected profit of entry, $E\left(\left(1-t_{\Pi}\right)\left[\alpha l\left(\alpha\right)-w\left(\alpha\right)\right]\right)$. The measure of entrants decreases and the exit threshold declines.

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